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Unparticles and Holographic Renormalization Group

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Abstract

We revisit the unparticle interactions and propagators from the AdS-CFT point of view, and we show how the contact terms and their renormalization group flow appear in the context of the holographic renormalization. We study both vector unparticles and unfermions, uncovering the relevant boundary conditions and renormalization group flows.

1 Introduction

Unparticles have very peculiar properties compared with ordinary particles. In his pioneering work, Georgi [1] defined unparticles as the "(approximately) scale invariant field theory that weakly couples with the standard model sector". The most important properties of unparticles is its scale invariance. The scale invariance might be imposed around the electro-weak energy scale, where we hope to find new physics in near-future experiments. The approximate scale invariance means that the scale invariance might (or might not) be broken at much higher (or lower) energy scale than the energy scale E that we would like to observe the unparticle.

As a simple example of the unparticle sector, Georgi considered Bank-Zaks (BZ) type conformal field theory (CFT) [2], which is defined by QCD with many massless fundamental fermions. Below the dynamical scale $\Lambda_{\mathcal{U}}$ of QCD, the theory is approximately conformal. If we introduce masses for the fermions, the conformal invariance would be broken at energy scale lower than $\Lambda_{\not{\mathcal{U}}}$. The approximate scale invariance demands the inequality $\Lambda_{\not{\mathcal{U}}} \ll E \ll \Lambda_{\mathcal{U}}$. Another important scale in unparticle physics is the mass scale $M_{\mathcal{U}}$ of the messenger fields, at which an unparticle operator O_{UV} at ultraviolet (UV) couples with a standard model (SM) operator O_{SM} as $\frac{O_{SM}O_{UV}}{M_{\mathcal{U}}^k}$. Below the conformal scale $\Lambda_{\mathcal{U}}$, it becomes the effective coupling between the scale invariant field theory and the SM sector as $\frac{C_{\mathcal{U}}\Lambda_{\mathcal{U}}^{d_{UV}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}O_{SM}O_{\mathcal{U}}$, where $k = d_{UV} + d_{\mathcal{U}} - 4$ with d_{UV} and $d_{\mathcal{U}}$ being the scaling dimension of unparticle operator at UV and scale invariant fixed point respectively.

Notice that the scaling dimension of unparticle operators is very important because when $d_{\mathcal{U}}$ is large, the interaction may be too weak to be observed in nature. However, if we assume the conformal invariance in the unparticle sector, there is a severe unitarity bound for the scaling (= conformal) dimension of primary operators [3]:

$$d \geq j_1 + j_2 + 2 - \delta_{j_1 j_2, 0} \ , \tag{1}$$

where j_1 and j_2 are Lorentz spin of the operator. As first pointed out in [4] (see also [5]), this unitarity bound is neglected by many authors in the study of vector unparticles, including Georgi's original work.

In fact, the unparticle interaction $\frac{O_{SM}O_{UV}}{M_{\mathcal{U}}^k}$ might not be the dominant contribution to the standard model process in new physics. For instance, the contact term interaction $\frac{O_{SM}^2}{M_{\mathcal{U}}^{k'}}$ introduced at the same UV scale $M_{\mathcal{U}}$ could be the dominant piece. Indeed, in [5], it has been shown that such an interaction should result from the renormalization group (RG) flow of the unparticle operators. Denoting the new interaction as $\sqrt{B_1}\frac{O_{SM}O_{\mathcal{U}}}{M_{\mathcal{U}}^k} + B_2\frac{O_{SM}^2}{M_{\mathcal{U}}^{k'}}$, they have shown that the Callan-Symanzik equation gives

$$\left(\frac{\partial}{\partial \log \mu} + \beta(g)\frac{\partial}{\partial g}\right) B_i = \gamma_{ij}(g)\, B_j \ , \tag{2}$$

where γ_{ij} are the anomalous dimension matrix. The solution to the RG equation can be obtained as

$$\begin{aligned} B_1(\mu) &= \left(\frac{\mu}{\Lambda_{\mathcal{U}}}\right)^{\gamma_{11}(g_*)} B_1(\Lambda_{\mathcal{U}}) && (3) \\ B_2(\mu) &= B_2(\Lambda_{\mathcal{U}}) + \frac{\gamma_{12}(g_*)}{\gamma_{11}(g_*)}\left[\left(\frac{\mu}{\Lambda_{\mathcal{U}}}\right)^{\gamma_{11}(g_*)} - 1\right] B_1(\Lambda_{\mathcal{U}})\,, && (4) \end{aligned}$$

where g_* is the non-trivial IR fixed point associated with the conformal sector.

Note that as discussed in [5], the ratio between the contribution from the unparticle exchange and the contact term can be computed as

$$\frac{A_{\text{unparticle}}}{A_{\text{contact}}} = \frac{B_2^2}{\sqrt{B_1}}\left(\frac{E}{M_{\mathcal{U}}}\right)^2\left(\frac{E}{\Lambda_{\mathcal{U}}}\right)^{2(d-3)} \ . \tag{5}$$

Obviously, for $E < \Lambda_{\mathcal{U}} < M_{\mathcal{U}}$ and vector unparticles with scaling dimension $d_V \geq 3$ (as required by unitarity), the unparticle exchange is naturally suppressed.

In this article, we will reproduce (4) by AdS-CFT correspondence. We will also show how the contact terms and their RG flow appear in the context of the holographic renormalization group. This requires a careful treatment of the boundary terms, which is sometimes neglected in the string theory literatures. As we will see, the boundary terms in the AdS-CFT generate the contact term interaction in the CFT and eventually lead to the effective standard model coupling $B_2\frac{O_{SM}^2}{M_{\mathcal{U}}^{k'}}$. The holographic renormalization group equation will give the counterpart of (4) in the CFT sector.

2 Vector Unparticles Revisited

One interesting theoretical approach to unparticle physics is to use AdS-CFT correspondence [6, 7, 8, 9, 10].[1] The basic statement of the AdS-CFT correspondence is that a strongly coupled conformal field theory can be analysed by a weakly coupled gravitational theory on AdS space. Although there is no known way to represent the gravity dual for SQCD (or Banks-Zaks theory), many other non-trivial superconformal field theories can be analysed from gravity.

It is rather trivial to see that both theories possess the same symmetry: on the CFT side, we have conformal $SO(2,4)$ symmetry while the AdS space has an isometry group given by $SO(2,4)$. In particular, under this correspondence, the AdS global energy (Hamiltonian) corresponds to the conformal dimension of CFT operators. In the following, we mainly consider the AdS space in the Poincare coordinate

$$ds^2_{AdS} = \frac{dz^2 + dx^\mu dx_\mu}{z^2} \ , \tag{6}$$

where the radial direction z corresponds to the energy scale of the CFT.

In addition to this kinematical correspondence, AdS-CFT predicts a dynamical relation (known as GKPW relation [11, 12]) between the generating functions of the CFT correlation functions and the path integral for the gravitational theory with fixed boundary condition:

$$Z_{AdS}[A_{0,\mu}] = \int_{A_M|_{boud}=A_{0,\mu}} \mathcal{D}A_M \exp(-I[A_M]) \equiv Z_{CFT}[A_{0,\mu}] = \left\langle \exp(\int d^4x A_{0,\mu} O^\mu) \right\rangle \ , \tag{7}$$

where $A_{0,\mu}$ is a suitably defined boundary value of the 5-dimensional vector field A_M and O^μ is the corresponding source current in the CFT. Later, we will use this relation to compute the unparticle propagator.

The unparticle hidden sector is not an idealistic CFT, however. At least we need (non-conformal) coupling between the hidden sector and the SM sector. We may also

[1] In this section, we would like to assume conformal invariance rather than mere scale invariance. The geometric description with only scale invariance is an interesting direction but it is not well understood. Maybe there is a geometrical way to prove or disprove the equivalence between conformal invariance and scale invariance in higher dimension.

want to introduce IR cut-off (or relevant deformation) below the electro-weak scale. In the AdS-CFT language, this field theory cut-off can be understood as a modification of the geometry at UV (or IR). We can introduce UV brane at $z = z_{UV} = \frac{1}{M_{\mathcal{U}}} = \epsilon$ to mimic the coupling to the SM sector. The IR cut-off can be also introduced by capping off the geometry at $z = z_{IR} = \frac{1}{\Lambda_{\mathcal{U}}}$. The construction is much like the Randall-Sundrum scenario (see e.g. [13]); and it is known as "unparticle deconstruction" [6].

In this section, we will revisit the vector unparticle propagator from the AdS-CFT point of view by focusing on the contact term interactions whose importance was emphasized in [5]. The contact terms are neglected in most applications of AdS-CFT because one can remove them by local counter terms in the boundary action. They are important, however, because they will affect the unparticle physics by introducing effective higher dimensional operators such as

$$L_{\text{eff}} = C_0 j_\mu j^\mu + C_1 j_\mu \partial^2 j^\mu + C_2 (\partial^\mu j_\mu)^2 + \cdots \tag{8}$$

in the standard model Lagrangian (j_μ is a current in the standard model).

The correspondence between the contact terms in the CFT and the higher dimensional interactions in the standard model Lagrangian can be understood as follows. We assumed the interaction $\frac{O_{SM} O_{UV}}{M_{\mathcal{U}}^k}$, so if we have a contact term interaction between $O_{\mathcal{U}}$, then the perturbative expansion of the standard model-unparticle interaction generates $\int d^4y \, O_{SM}(y) O_{SM}(x) \langle O_{\mathcal{U}}(y) O_{\mathcal{U}}(x) \rangle$, which will yield us a higher derivative interaction $B_2 \frac{O_{SM}^2}{M_{\mathcal{U}}^{k'}}$ in (8) from the contact terms such as $C_0 \delta(x) + C_1 \partial^2 \delta(x) + \cdots$ (in addition to the conventional standard model-unparticle interaction $\sqrt{B_1} \frac{O_{SM} O_{\mathcal{U}}}{M_{\mathcal{U}}^k}$). With this correspondence, we can identify the coupling constants C_i in (8) as the coefficients appearing in the contact term of the correlation functions in the CFT. We will show that the natural RG flow generates such terms completely in agreement with the field theory discussion [5]. From the higher dimensional brane scenario perspective, our prescription provides a natural way to understand the evolution of the boundary local counter terms under the RG flow.

First of all, the action for the 5-dimensional massive vector (5d Proca action) is given by [14]

$$I = \int d^5x\sqrt{g}\left(\frac{1}{4}F_{MN}F^{MN} + \frac{1}{2}m^2 A_M A^M\right), \tag{9}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. This leads to the equation of motion (Proca equation)

$$\nabla_M F^{MN} - m^2 A^N = 0\,. \tag{10}$$

By taking the divergence of the Proca equation[2], we obtain the divergence free condition

$$\nabla_M A^M = 0\,. \tag{11}$$

This action can be evaluated by the boundary data $\tilde{A}_{\epsilon,\mu}(k)$, which is the Fourier transform of the Dirichlet boundary value of the field $\tilde{A}_\mu(k)$ at $z = \epsilon$:

$$\begin{aligned} I \;=\; & \epsilon^{-4}\,\frac{d-3}{2}\int\frac{d^4k}{(2\pi)^4}\,\tilde{A}_{\epsilon,\mu}\tilde{A}_{\epsilon,\mu} \\ & -\frac{\epsilon^{-2}}{4}\,\frac{\Gamma(d-3)}{\Gamma(d-2)}\int\frac{d^4k}{(2\pi)^4}\,k^2\,\tilde{A}_{\epsilon,\mu}\left(-\delta_{\mu\nu} + \frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right)\tilde{A}_{\epsilon,\nu} \\ & -\frac{\epsilon^{2(d-4)}}{4^{d-2}}\,\frac{\Gamma(3-d)}{\Gamma(d-2)}\int\frac{d^4k}{(2\pi)^4}\,(k^2)^{d-2}\,\tilde{A}_{\epsilon,\mu}\left(-\delta_{\mu\nu} + \frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right)\tilde{A}_{\epsilon,\nu} + \cdots, \end{aligned} \tag{12}$$

where higher derivative terms with higher order ϵ is neglected. For later purposes, however, we have incorporated the contact terms neglected in [14].[3]

The mass m in the 5d-bulk space is related to the conformal dimension d of the dual operator as $d = 2 + \sqrt{1+m^2}$ [14]. Generalizing the discussion in [12], one can easily see that for the vector particle, the stability bound is $m^2 \geq 0$, corresponding to the unitarity bound for the vector unparticle $d_V \geq 3$. The necessity of the unitarity bound can also be seen as the requirement of the (Euclidean) non-normalizabity of the wave under the inner product $\int d^5x\sqrt{g}\,g^{MN}A_M A_N$ with $A_\mu(z) \sim z^{4-d}$ near $z \sim 0$.

The third line in (12), which is in general non-analytic, will reproduce the CFT two-point function [5] (up to a normalization factor c)

[2]When $m^2 = 0$, this is nothing but a Lorenz gauge condition, but for $m^2 \neq 0$ it follows from the equation of motion.

[3]The conformal invariance does not fix the structure of the contact terms, so this is the reason why they are often neglected in the literatures of AdS-CFT correspondence. Here, we show that the boundary counter terms play an important role to determine the contact terms and their RG-flow.

$$
\begin{aligned}
\langle O_\mu(x) O_\nu(0)\rangle &= \frac{c}{2\pi^2}\frac{\delta_{\mu\nu} - 2x_\mu x_\nu/x^2}{(x^2)^d} \\
&= c\,\frac{(d-1)\Gamma(2-d)}{4^{d-1}\Gamma(d+1)}\int \frac{d^4k}{(2\pi)^4}\, e^{ikx}(k^2)^{d-2}\left(\delta_{\mu\nu} - \frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right) \qquad (13)
\end{aligned}
$$

from the AdS-CFT prescription [11, 12] as shown in (7) together with a suitable analytic continuation in the Fourier integral. This is achieved by specifying the boundary data $\tilde{A}_{0,\mu} = \lim_{\epsilon\to 0}\tilde{\mathcal{A}}_{\epsilon,\mu}$ with the normalized field $\tilde{\mathcal{A}}_{\epsilon,\mu} \equiv \epsilon^{d-4}\tilde{A}_{\epsilon,\mu}$.

In contrast, the first line and the second line in (12) are not dictated by the conformal invariance but they give contact terms. At a given ϵ, one can always eliminate such contact terms by adding the boundary counter terms as

$$
\begin{aligned}
\delta S_{bound} &= \epsilon^{-2(4-d)}\int \frac{d^4k}{(2\pi)^4}\left(c_0\tilde{A}_{\epsilon,\mu}\tilde{A}_{\epsilon,\mu} + k^2\,\tilde{A}_{\epsilon,\mu}\left(c_1\delta_{\mu\nu} + c_2\frac{k_\mu k_\nu}{k^2}\right)\tilde{A}_{\epsilon,\nu}\ + \cdots\right) \\
&= \int \frac{d^4k}{(2\pi)^4}\left(c_0\tilde{\mathcal{A}}_{\epsilon,\mu}\tilde{\mathcal{A}}_{\epsilon,\mu} + k^2\,\tilde{\mathcal{A}}_{\epsilon,\mu}\left(c_1\delta_{\mu\nu} + c_2\frac{k_\mu k_\nu}{k^2}\right)\tilde{\mathcal{A}}_{\epsilon,\nu} + \cdots\right), \qquad (14)
\end{aligned}
$$

which are localized on the UV-brane. However, what we would like to study here is the RG flow of the contact terms. In other words, we would like to investigate the cut-off dependence of the contact terms in the AdS-CFT setup.

We find that it is natural to introduce the cut-off dependence on the boundary counter term by parameterizing $c_0 = \tilde{C}_0\,\epsilon^{4-2\Delta_0}$ and $c_{1,2} = \tilde{C}_{1,2}\,\epsilon^{6-2\Delta_0}$, where we have introduced the "naive dimension" Δ_0 of the current operator under consideration. The point is that the RG equation in [5] involves the "anomalous" dimension which is only defined by comparing the actual dimension of certain operators with a reference value (say, a UV free theory). In fact, they made an assumption that they normalize their operators with respect to the UV free theory. Correspodingly, the cut-off dependence introduced here is normalized so that for the free field current interaction (i.e. $\Delta_0 = 3$), $c_{1,2}$ are cut-off independent and dimensionless. Once we have determined to evaluate the anomalous dimension of CFT operators with respect to the free field theory by utilizing the same convention used in [5], the vanishing cut-off dependence for $\Delta_0 = 3$ is fixed by definition. Simple dimensional analysis also determines other cut-off dependence like c_0. Now, with different cut-offs, we have the relation:

$$
\begin{aligned}
C_0(\epsilon) &= \tilde{C}_0\, \epsilon^{4-2\Delta_0} \left(1 - \left(\frac{\tilde{\epsilon}_0}{\epsilon} \right)^{\gamma} \right) , && (15) \\
C_1(\epsilon) &= \tilde{C}_1\, \epsilon^{6-2\Delta_0} \left(1 - \left(\frac{\tilde{\epsilon}_1}{\epsilon} \right)^{\gamma} \right) , && (16) \\
C_2(\epsilon) &= \tilde{C}_2\, \epsilon^{6-2\Delta_0} \left(1 - \left(\frac{\tilde{\epsilon}_2}{\epsilon} \right)^{\gamma} \right) , && (17)
\end{aligned}
$$

where we have introduced the anomalous dimension $\gamma = 2(d - \Delta_0)$. $\tilde{\epsilon}_i$ denotes the scale at which the boundary counter terms cancel the bulk contributions, and they can be different for different i in principle. It is easy to see that C_1 in (16) is equivalent to B_2 in (4) by the identifications $\Delta_0 = 3$ and $\epsilon \sim \frac{1}{\mu}$. In this way, we have shown how AdS-CFT correspondence also predicts the appearance of the contact terms and their evolution.

Several comments are in order:

- The choice of naive dimension $\Delta_0 = 3$ is natural because $\tilde{A}_{\epsilon,\mu}$ couples to the vector operator O^μ, and the typical (actually the lowest dimensional) free field vector operator has dimension 3 such as $\phi^\dagger \partial_\mu \phi$ or $\bar{\psi}\gamma_\mu\psi$.

- Unlike the claim ($C_0 = C_1$) in [5], C_0 and C_1 are not a-priori related though we could always relate them as a boundary condition at the cut-off. This difference is due to the fact that they implicitly assumed the simplest weakly coupled messengers that propagate between the unparticle sector and the standard model sector. For a more general strongly coupled mediation, the condition ($C_0 = C_1$) will be generically violated.

- When d is an integer, the distinction between the boundary counter terms (contact terms) and the bulk term is less clear because the propagator is analytic in k. This is somehow related to the artificial divergence of some unparticle amplitudes at integer value of d appearing in the literatures [5]. It simply suggests that the normalization of the operator is not good: we can always remove the divergence by the counter term or proper choice of the renormalized coupling constant.

- Since the AdS gravity dual does not know anything about the "anomalous" dimension but only knows the "actual" conformal dimensions, the introduction of the naive dimension as a regularization (boundary counter term) is necessary. Our prescription

is the most natural one in the sense that it is in complete agreement with the field theory. In principle, we could embed the whole system inside an "asymptotically free field dual" of the gravity theory to discuss the anomalous dimensions and operator evolution without using somewhat artificial boundary counter terms. Our prescription, however, should be an effective way to implement this hypothetical procedure because there is no known simple gravity dual for asymptotically free field theories.

3 Unfermions

A similar construction is also possible for the Dirac field (unfermion) [9]. The 5d action is given by [14]

$$\int d^5x\sqrt{g}\,\bar{\psi}(\not{D}-m)\psi + G\int d^4x\sqrt{h}\,\bar{\psi}\psi\,, \tag{18}$$

which is supplemented with a surface term [15] with an undetermined coefficient G.

The bulk Dirac equation

$$(\not{D}-m)\psi = 0 \tag{19}$$

gives the relation between the left-mover ψ_ϵ^+ and right-mover ψ_ϵ^- at the boundary $z=\epsilon$, and the boundary action can be determined solely from the boundary term. The action is

$$I = i\,G\,\epsilon^{-3}\,\frac{\Gamma(d_F-\frac{5}{2})}{\Gamma(d_F-\frac{3}{2})}\int\frac{d^4k}{(2\pi)^4}\left(\bar{\psi}_\epsilon^+ k_\mu\gamma^\mu\psi_\epsilon^- - \frac{\Gamma(d_F-\frac{1}{2})}{\Gamma(d_F-\frac{3}{2})}\left(\frac{k\,\epsilon}{2}\right)^{2\,d_F-5}\bar{\psi}_\epsilon^+ k_\mu\gamma^\mu\psi_\epsilon^- + \cdots\right), \tag{20}$$

where $d_F = m+2$ is the scaling dimension of the unfermions [14].[4]

The second term in (20) will generate the correct unfermion propagator as follows. Let χ^+ and $\bar{\chi}^-$ be the boundary spinors which couple to $\bar{\psi}_0^+$ and ψ_0^- respectively, where

$$\bar{\psi}_0^+ = \lim_{\epsilon\to 0}\epsilon^{\,d_F-4}\,\bar{\psi}_\epsilon^+ \quad \text{and} \quad \psi_0^- = \lim_{\epsilon\to 0}\epsilon^{\,d_F-4}\,\psi_\epsilon^-\,. \tag{21}$$

[4]We concentrate on $m\geq 1/2$ here for simplicity. The case $m\leq -1/2$ can be treated similarly but with the roles of ψ_ϵ^+ and ψ_ϵ^- exchanged [14]. See also [9] for $|m|<1/2$.

Then, from the AdS-CFT correspondence:

$$\exp(-I_{AdS}) \equiv \left\langle \exp\left(\int d^4x \, \left(\bar{\chi}^- \, \psi_0^- + \bar{\psi}_0^+ \, \chi^+ \right) \right) \right\rangle \,, \tag{22}$$

the unfermion propagator after Fourier transformation to the coordinate space will be given by [14]

$$\left\langle \chi^+(x) \, \bar{\chi}^-(y) \right\rangle = \frac{2\,G}{\pi^2} \frac{\Gamma(d_F + \frac{1}{2})}{\Gamma(d_F - \frac{3}{2})} \frac{\gamma_\mu (x^\mu - y^\mu)}{|x-y|^{2(d_F + \frac{1}{2})}} \,. \tag{23}$$

On the other hand, the first term in (20) gives the contact term interaction $\not{\partial}\,\delta(x)$.[5] Again, as is the case with the vector unparticle, the contact term can be removed (at a given ϵ) by adding the boundary counter term as

$$\begin{aligned} \delta S_{bound} &= \epsilon^{-2(4-d_F)} \int \frac{d^4k}{(2\pi)^4} \, \left(c_1 \, \bar{\psi}_\epsilon^+ k_\mu \gamma^\mu \psi_\epsilon^- + \cdots \right) \\ &= \int \frac{d^4k}{(2\pi)^4} \, \left(c_1 \, \bar{\psi}_0^+ k_\mu \gamma^\mu \psi_0^- + \cdots \right) \,, \end{aligned} \tag{24}$$

which are localized on the UV-brane. A natural way to introduce the cut-off dependence of the boundary counter-term is again characterized by the naive dimension Δ_0 as $c_1 = \tilde{C}_1 \, \epsilon^{5-2\Delta_0}$. Therefore, the RG evolution of the contact term is given by

$$C_1(\epsilon) = \tilde{C}_1 \, \epsilon^{5-2\Delta_0} \left(1 - \left(\frac{\tilde{\epsilon}_1}{\epsilon} \right)^\gamma \right) \,, \tag{25}$$

where we have introduced the anomalous dimension $\gamma = 2(d_F - \Delta_0)$.

The cut-off dependence (25) is consistent with the Callan-Symanzik equation for the unfermion interaction. In fact, C_1 will reproduce the solution to the Callan-Symanzik equation by the identifications $\Delta_0 = \frac{5}{2}$ and $\epsilon \sim \frac{1}{\mu}$.

As a remark, it is important to note that the non-derivative contact term $\bar{\psi}\psi$ is *not* generated through this regularization procedure. Thus, the contact interactions $\bar{O}_{\rm SM} O_{\rm SM}$

[5]The use of this unfermion contact (or non-local) interaction is not so clear because there is no fermionic gauge singlet in the standard model. Gauged unparticle [16] would be possible, but it is highly constrained by experiments.

and $\bar{O}_{\rm SM}\,\partial\!\!\!/\,O_{\rm SM}$ are *not* related at all. This should be contrasted with the vector unparticle, where the non-derivative contact term is also introduced.[6]

4 Conclusions

In this article, we revisited the unparticle interactions and propagators from the AdS-CFT point of view. We studied both vector unparticles and unfermions, revealing the relevant boundary conditions and RG flows. Our focus is on the contact terms whose importance was emphasized in [5], but have been ignored by previous studies of unparticles. We have shown how the holographic RG flow can generate such contact terms and their evolution. This construction is the most natural one in the sense it is in complete agreement with the field theory discussion [5]. Our prescription also provides a natural way to understand the evolution of the boundary local counter terms under the RG flow.

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